Empirical Study of the Effect of Including Skewness and Kurtosis in Black-Scholes Option Pricing Formula on S&P CNX Nifty Index Options

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The most popular model for pricing options, both in financial literature as well as in practice has been the Black-Scholes model. In spite of its widespread use, the model appears to be deficient in pricing deep-in-the-money and deep-out-of-the-money options using statistical estimates of volatility. This limitation has been taken into account by practitioners using the concept of implied volatility. Many improvements to the Black-Scholes formula have been suggested in academic literature for addressing the issue of volatility smile. This paper studies the effect of using a variation of the Black-Scholes model (suggested by Corrado and Sue incorporating non-normal skewness and kurtosis) to price call options on S&P CNX Nifty. The results strongly suggest that the incorporation of skewness and kurtosis into the option pricing formula yields values much closer to market prices. Based on this result and the fact that this approach does not add any further complexities to the option pricing formula, it is suggested that this modified approach should be considered as a better alternative.

JEL Classification: G13.

Introduction

The most popular model for pricing options, both in financial literature as well as in practice, has been the one developed by Fischer Black and Myron Scholes. In spite of its widespread use, the model appears to be deficient in pricing deep-in-the-money and deep-out-of-the-money options using statistical estimates of volatility. This limitation has been taken into account by the practitioners using the concept of implied volatility. Implied volatility is the value of statistical volatility needed to be used in the standard Black-Scholes pricing formula for a given strike price to yield the market price of that option. The value of implied volatility for different strike prices should theoretically be identical, but is usually seen to vary in the markets. In most markets across the world, it has been observed that the implied volatilities for different strike prices form a pattern of either a 'smile' or a 'skew'. Theoretically, since volatility is a property of the underlying asset it should be predicted by the pricing formula to be identical for all the derivatives based on that same

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asset. Hull (1993) and Nattenburg (1994) have attributed the volatility smile to the non-normal skewness and kurtosis of stock returns, which is contrary to the assumption of Black-Scholes model.

Many improvements to the Black-Scholes formula have been suggested in literature for addressing the issue of volatility smile. Corrado and Sue (1996a) have extended the Black-Scholes formula to account for non-normal skewness and kurtosis in stock return distributions. Their assumption is that if the volatility smile is due to non-normal skewness and kurtosis of the distribution of asset returns, this could be removed if the effects of these deviations are included in the pricing formula. The method suggested by them for incorporation of this deviation is based on the fitting of the first four moments of stock return distribution on to a pattern of empirically observed option prices. The values of implied volatility, implied skewness and implied kurtosis have been estimated by minimizing the error between predicted and actual market option prices for all available strike prices.

In this paper we address the volatility smile pattern as observed in the NSE Nifty index options. Section 2 discusses the current literature on this topic. Sections 3 and 4 present the applied model and the implementation details. Section 5 presents the results obtained. Finally, the paper presents the conclusion of this study and the managerial implications thereof.

2. Literature Review

In all major markets across the world, varying implied volatilities of options on the same underlying asset across different exercise prices and terms to maturity have been observed. In a recent study on NSE Nifty, Misra *et al.* (2006) have reported a significant volatility smile on Nifty options. The results of their study show that deep-in-the-money and deep-out-of-the-money options have higher volatility than at-the-money options, and the implied volatility of out-of-the-money call options is greater than in-the-money calls. Daily returns of NSE Nifty have been found to follow normal distribution with some skewness and kurtosis. These results suggest that the volatility smile observed in the case of NSE Nifty options can be explained to some measure by the observed skewness and kurtosis.

To incorporate the effects of non-normal skewness and kurtosis into the Black-Scholes option pricing formula, Hermite polynomials have been used to get an expansion of the probability density function adjusted for skewness and kurtosis. Usually, this series is called Gram-Charlier. For practical purposes, only the first few terms of this expansion are taken into consideration. The resulting truncated series may be viewed as the normal probability density function multiplied by a polynomial that accounts for the effects of departure from normality. The Gram-Charlier series uses the moments of the real distribution. The Edgeworth series is similar to Gram-Charlier but uses cumulants instead of moments. Although both the series are equivalent, for computational purposes the Gram-Charlier series seems to perform better than the Edgeworth series (Johnson *et al.*, 1994).

This approach was introduced in financial economics by Jarrow and Rudd (1982), and it has been applied by Madam and Milne (1991), Longstaff (1995), Abken *et al.* (1996a and 1996b), Backus *et al.* (1997), Brenner and Eom (1997), Corrado and Sue (1997) and Knight and Satchell (1997).

Jarrow and Rudd (1982) proposed a semi-parametric option pricing model to account for observed strike price biases in the Black-Scholes model. They derived an option pricing formula from the Edgeworth expansion of the lognormal probability density function to model the distribution of stock prices. Corrado and Sue (1996a) have adapted this extension developed by Jarrow and Rudd to extend the Black-Scholes formula to account for non-normal skewness and kurtosis in stock returns. While following the same methodology, they used a Gram-Charlier series expansion of the normal probability density function to model the distribution of stock log prices. This method fits the first four moments of a distribution to a pattern of empirically observed option prices. The mean of this distribution is determined by option pricing theory, but an estimation procedure is employed to yield implied values for the standard deviation, skewness and kurtosis of the distribution of stock index prices. We have used the extended formula adapted by Corrado and Sue (1996b) for the NSE Nifty index options to address the volatility smile reported.

3. Model Applied

Corrado and Sue (1996a) have developed a method to incorporate effects of non-normal skewness and kurtosis of asset returns into an expanded Black-Scholes option pricing formula. Brown and Robinson (2002) suggested a correction to this approach which was incorporated in Corrado and Sue (1997). Their method adopts a Gram-Charlier series expansion of the standard normal density function to yield an option price formula which is the sum of Black-Scholes option price plus two adjustment terms for non-normal skewness and kurtosis. Specifically, the density function $g(z)$ defined below accounts for non-normal skewness and kurtosis, denoted by μ_3 and μ_4 respectively, where $n(z)$ represents the standard normal density function.

$$g(z) = n(z)\left[1 + \frac{\mu_3}{3!}\left(z^3 - 3z\right) + \frac{\mu_4 - 3}{4!}\left(z^4 - 6z^2 + 3\right)\right]$$

where,

$$z = \frac{\ln\left(S_t / S_0\right) - \left(r - \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

and

S_0 is the current asset price,

S_t is the stochastic asset price at time t,

r is the risk-free rate of interest,

σ is the standard deviation of the returns for the underlying asset.

In the formula mentioned above, skewness μ_3 and kurtosis μ_4 have been explicitly used in the density function $g(z)$, in a functional form. For the normal distribution curve the values of these coefficients are: skewness, $\mu_3 = 0$ and kurtosis, $\mu_4 = 3$.

Using the function $g(z)$, the value of the theoretical call price as the present value of the expected payoff at option expiration was found out to be:

$$C = e^{-rt} \int_K^{\infty} (S_t - K) g\left(z(S_t)\right) dz(S_t)$$

where,

K is the strike price.

$$z(S_t) = (\log S_t - \mu)/\sigma\sqrt{t}$$

$$\mu = \log S_0 + \left(r - \frac{\sigma^2}{2}\right)t$$

The above integral can be evaluated using the Gram-Charlier density expansion, and the evaluated option price is denoted as C_{GC} while the Black-Scholes option price formula is denoted as C_{BS}.

$$C_{BS} = S_0 N(d) - Ke^{-rt} N\left(d - \sigma\sqrt{t}\right)$$

$$d = \frac{\ln\left(S_0/K\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$C_{GC} = C_{BS} + \mu_3 Q_3 + (\mu_4 - 3) Q_4$$

where,

$$Q_3 = \frac{1}{3!} S_0 \sigma\sqrt{t}\left(\left(2\sigma\sqrt{t} - d\right)n(d) + \sigma^2 t N(d)\right)$$

$$Q_4 = \frac{1}{4!} S_0 \sigma\sqrt{t}\left(d^2 - 1 - 3\sigma\sqrt{t}\left(d - \sigma\sqrt{t}\right)n(d) + \sigma^3 t^{3/2} N(d)\right)$$

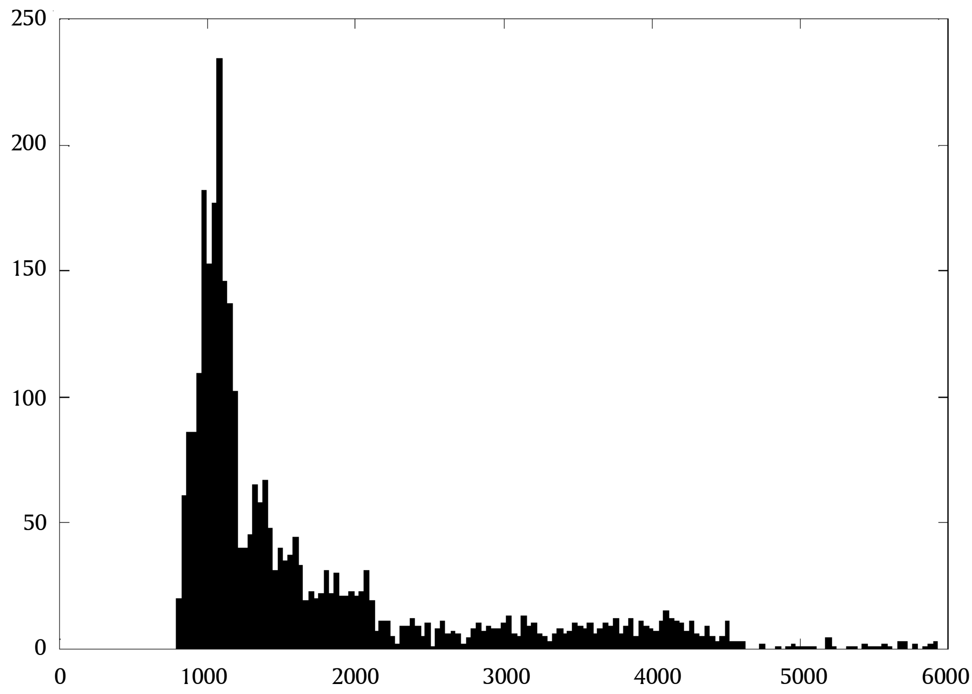
In the adjusted formula, the terms $\mu_3 Q_3$ and $(\mu_4 - 3)Q_4$, measure the effects of the non-normal skewness and kurtosis on the option price C_{GC}.

For this study, we have used both the Black-Scholes formula and the modified formula as suggested by Corrado and Sue (1996b) to calculate the option prices. Thereafter, the error in both the cases has been calculated and tested for statistically significant difference using the paired t-test.

3.1 Data Used

We started the study by testing the fact that the Nifty closing values do not follow a lognormal distribution and that Nifty returns do not conform to the normal distribution. Figure 1 shows the frequency plot of Nifty closing values.

Figure 1: Frequency Plot of NSE Nifty Daily Closing Values (3/11/95-13/11/07)



For examination purpose, we computed the mean, standard deviation, skewness and kurtosis of the daily and weekly NSE Nifty returns since its inception. The values are reported in Table 1. The distribution of the returns is shown in Figures 2 and 3 and is superimposed on the normal distribution of identical mean and variance for better comparison.

Table 1: NSE Nifty Returns Data

	NSE Nifty Daily Returns	NSE Nifty Weekly Returns
Mean	0.0006	0.0023
Std. Deviation	0.0160	0.0330
Skewness	–0.3168	–0.3334
Kurtosis	4.4682	1.7953

Figure 2: Frequency Plot of NSE Nifty Daily Returns (3/11/95-13/11/07)

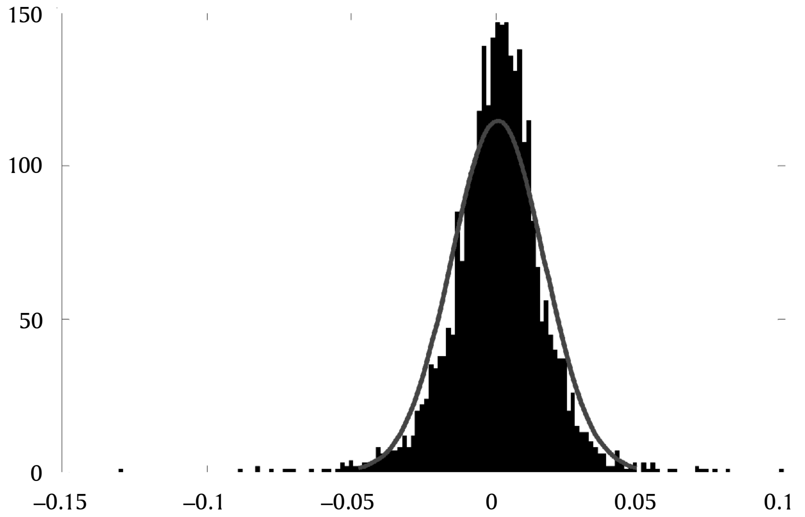
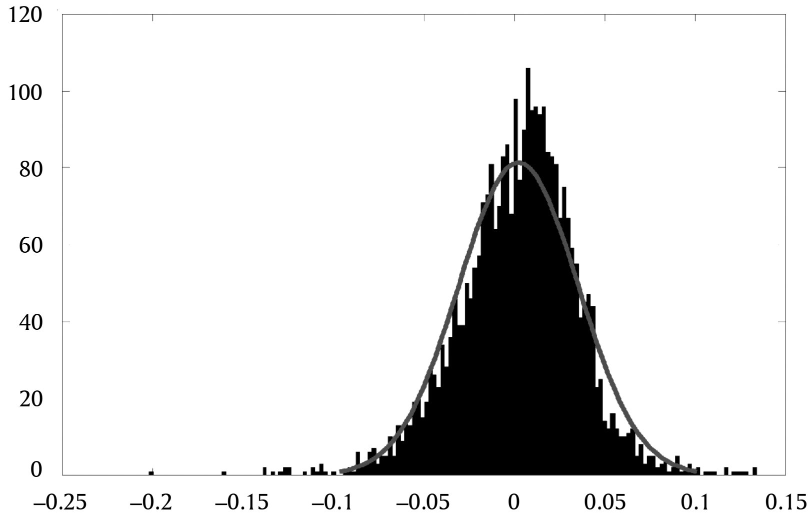


Figure 3: Frequency Plot of NSE Nifty Weekly Returns (3/11/95-13/11/07)



From the analysis, it can be observed that the distribution of Nifty returns has a negative skew and positive kurtosis as expected and reported for most markets across the world. After confirming significant skewness and kurtosis in the returns, the Black-Scholes formula and the skewness and kurtosis adjusted formula have been applied for prediction of option prices.

Thereafter, the analysis was based on call prices of Nifty Options (European type) for the period of about three months from August 1, 2007 to October 24, 2007. Data for thinly traded options (less than 100 contracts on a given day) were excluded from the study.

4. Methodology

4.1 The Black-Scholes Option Pricing Formula

The four parameters of Black-Scholes option pricing formula namely, stock price, strike price, time to option maturity and the risk-free interest rate have been directly observed from the market. The daily Mumbai Inter Bank Offer Rate (MIBOR) has been taken as the risk-free interest rate. Another input to the formula was the standard deviation of the stock price. This should theoretically be identical for options of all strike prices because the underlying asset is the same in each case. But, since this is not directly observable, it has been estimated using the following method:

Using option prices for all contracts within a given maturity series observed on a given day, we estimated a single implied standard deviation to minimize the total error sum of squares between the predicted and the market prices of options of various strike prices. This has been calculated using Microsoft Excel Solver function by minimizing the following function, by iteratively changing the implied standard deviation.

$$\min_{BSISD} \sum_{j=1}^{N} \left[C_{OBS,j} - C_{BS,j}(BSISD) \right]^2$$

where *BSISD* stands for the Black-Scholes Implied Standard Deviation

After all the input variables for the model are obtained, they are used to calculate theoretical option prices for all strikes within the same maturity series for the following day. Thus, theoretical option prices for a given day are based on a prior-day, out-of sample implied standard deviation estimated. We then compare these theoretical prices with the actual market prices observed on that day.

4.2 Skewness and Kurtosis Adjusted Black-Scholes Option Pricing Formula

Next, we assessed the skewness and kurtosis adjusted Black-Scholes option pricing formula developed by Jarrow and Rudd (1982) using an analogous procedure. Specifically, on a given day we estimate a single implied standard deviation, a single skewness coefficient, and a single excess kurtosis coefficient by minimizing once again the error sum of squares represented by the following formula:

$$\min_{ISD,ISK,IKT} \sum_{j=1}^{N} \left[C_{OBS,j} - \left(C_{BS,j}(ISD) + ISK * Q_3 + (IKT - 3) * Q_4 \right) \right]^2$$

where *ISD*, *ISK* and *IKT* represent estimates of the implied standard deviation, implied skewness and implied kurtosis parameters based on *'N'* price observations.

We then used these three parameter estimates as inputs to the Jarrow-Rudd formula to calculate the theoretical option prices corresponding to all option prices within the same maturity series observed on the following day. Thus, these theoretical option prices for a given day are based on prior-day, out-of-sample implied standard deviation, skewness, and excess kurtosis estimates. We then compared these theoretical prices with the actual market prices observed on that day.

Hypothesis: The total error in prediction of option prices for various strike prices by the modified Black-Scholes method is less than that by the original Black-Scholes method.

4.3 Comparison

The theoretical option prices thus generated using the two approaches are then compared with their actual market prices. For comparison, we compute the Error Sum of Squares (ESS) for the two approaches by summing the square of the difference between the predicted and the actual prices. These two ESS are then compared for statistically significant difference using the paired *t*-test.

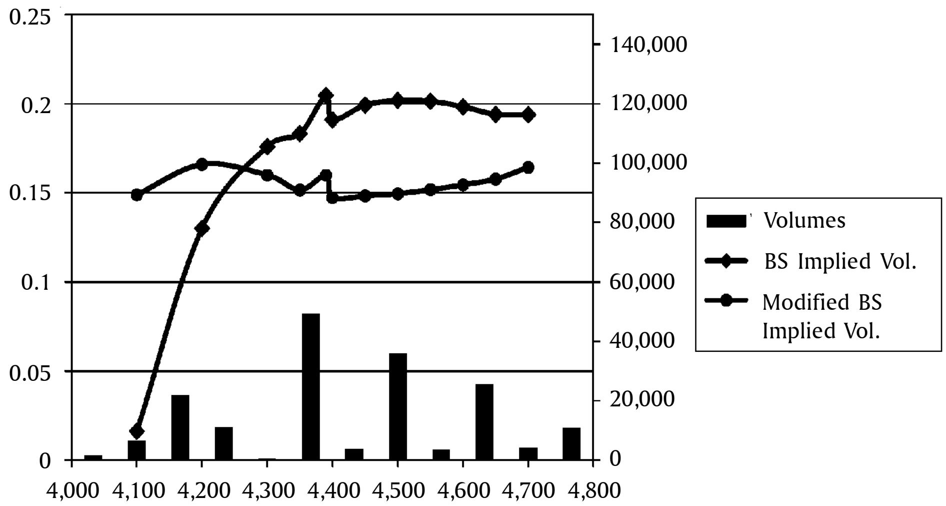
5. Results

The paired *t*-test of the samples of ESS, results in a *t*-value of 7.57 which overwhelmingly rejects the hypothesis that the errors in prediction of option prices by the two methods are not significantly different.

We also performed the comparison test independently for options of all four maturities. The *t*-values in case of each of the four separate tests were greater than the critical value at 95% confidence level. Hence, we see that the modified Black-Scholes formula appears to price Nifty options much closer to the actual market prices.

Figure 4 shows the implied volatilities calculated using the two methods. Black-Scholes (BS) implied volatilities are the usual volatilities required to be inserted into the BS formula so that it gives the market price of the option. For the modified BS method, the skewness and kurtosis have been kept constant and equal to that obtained upon reducing the total error in the pricing of options of all strikes for a given maturity for that day (as explained in

Figure 4: Implied Volatiliy Curve (BS and Modified BS) for Nifty Call Options on August 01, 2007 (Nifty Call Options Expiring on August 30, 2007)



Note: For the modified BS curve, skewness = 2.13 and kurtosis = –2.44.

methodology), and then the volatilities have been calculated as those required to be inserted into the modified BS formula so that it gives the market price of the option.

The volatility smile as observed for the BS model is significant, while for the modified BS model the implied volatility curve is almost flat. The detailed results are presented in Tables 2 and 3.

Table 2: Paired *t*-test Results (Combined for All Strike Prices)

t-test: Paired Two Sample for Means		
	BS Original	**BS Modified**
Mean	20598.156610000	2307.040835
Variance	342544637.200000000	10664366.580000
Observations	60	60
Pearson Correlation	0.024857351	–
Hypothesized Mean Difference	0	–
df	59	–
***t*-statistic**	**7.571032779**	–
$P(T<=t)$ one-tail	1.45726E-10	–
t-Critical one-tail	1.671093033	–
$P(T<=t)$ two-tail	2.91451E-10	–
t-Critical two-tail	2.000995361	–

Table 3: Paired *t*-test Results (Separately for Different Strike Prices)

t-test: Paired Two Sample for Means – August 30		
	BS Original	**BS Modified**
Mean	20340.744180000	1302.505601
Variance	424366615.500000000	1382374.835000
Observations	10	10
Pearson Correlation	–0.55641946	–
Hypothesized Mean Difference	0	–
df	9	–
***t*-statistic**	**2.829569097**	–
$P(T<=t)$ one-tail	0.009868142	–
t critical one-tail	1.833112923	–
$P(T<=t)$ two-tail	0.019736283	–
t critical two-tail	2.262157158	–

(Contd...)

Table 3: Paired *t*-test Results (Separately for Different Strike Prices) *(...contd)*		
***t*-test: Paired Two Sample for Means – September 27**		
	BS Original	**BS Modified**
Mean	30932.30180000	1982.878403
Variance	389184830.00000000	5497910.873000
Observations	20	20
Pearson Correlation	0.2606916	–
Hypothesized Mean Difference	0	–
df	19	–
***t*-statistic**	**6.72546979**	–
$P(T<=t)$ one-tail	9.958E-07	–
t critical one-tail	1.72913279	–
$P(T<=t)$ two-tail	1.9916E-06	–
t critical two-tail	2.09302405	–
***t*-test: Paired Two Sample for Means – October 25**		
	BS Original	**BS Modified**
Mean	18908.97069000	3382.171758
Variance	192781467.80000000	19322125.720000
Observations	20	20
Pearson Correlation	–0.13652606	–
Hypothesized Mean Difference	0	–
df	19	–
***t*-statistic**	**4.59090931**	–
$P(T<=t)$ one-tail	9.97638E-05	–
t critical one-tail	1.729132792	–
$P(T<=t)$ two-tail	0.000199528	–
t critical two-tail	2.09302405	–
***t*-test: Paired Two Sample for Means – November 29**		
	BS Original	**BS Modified**
Mean	3565.65045000	1809.639087
Variance	26532333.20000000	11932312.040000
Observations	10	10
Pearson Correlation	0.96883806	–
Hypothesized Mean Difference	0	–

(Contd...)

Table 3: Paired *t*-test Results (Separately for Different Strike Prices) *(...contd)*		
***t*-test: Paired Two Sample for Means – November 29**		
	BS Original	**BS Modified**
df	9	–
***t*-statistic**	**2.78084378**	–
$P(T<=t)$ one-tail	0.01068564	–
t critical one-tail	1.83311292	–
$P(T<=t)$ two-tail	0.02137127	–
t critical two-tail	2.26215716	–

Conclusion and Managerial Policy Implications

The results obtained from the analysis confirm our hypothesis that the modified Black-Scholes model as put forward by Corrado and Sue performs significantly better for NSE Nifty option prices. We also see that the calculation process in the modified model is not very different from that of the original Black-Scholes model. Since it does not add unnecessary complexity and still gives significantly better predictions of option prices, we recommend that this modified model should be looked at as a better alternative to the existing method.

This study also confirms that fitting of higher order moments of the distribution of returns is important, especially for options away from the money. Figure 4 graphically shows that the 'smile' of the implied volatility curve can be explained by the reduction in the implied skewness and kurtosis by using the modified BS formula. This study also suggests further work in this area. In this paper, we have calculated implied volatility, skewness and kurtosis based on today's data to predict tomorrow's prices. This can be extended to explore whether the modified approach gives significantly better prices for longer durations or not. A related study could be regarding comparison of returns achieved using trading strategies based on these two different models. It would be interesting to see if significant gains can be made using the modified Black-Scholes model over the original model. ✧

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Appendix A

Data on Volatility Smile							
Date	Expiry	Strike Price	BS Implied Volume	Modified BS Implied Volume	Volumes	BS Modified Implied Skew	BS Modified Implied Kurt
01-Aug-07	30-Aug-07	4,100	0.016200000	0.148910961	1,448	4.638195670	1.138337607
01-Aug-07	30-Aug-07	4,200	0.130055461	0.165977733	6,628	1.997287203	3.001311393
01-Aug-07	30-Aug-07	4,300	0.175867395	0.159943408	21,800	2.009064915	2.986649372
01-Aug-07	30-Aug-07	4,350	0.183203284	0.151586922	11,183	2.000371554	2.999023259
01-Aug-07	30-Aug-07	4,390	0.204689112	0.159854501	443	2.006943806	2.989818834
01-Aug-07	30-Aug-07	4,400	0.190901711	0.147323983	49,270	2.000230589	2.999689689
01-Aug-07	30-Aug-07	4,450	0.199143620	0.148267320	3,789	1.997030102	3.005560229
01-Aug-07	30-Aug-07	4,500	0.201871418	0.149495946	35,965	1.958001070	3.057268601
01-Aug-07	30-Aug-07	4,550	0.201336728	0.151756237	3,497	1.801023346	3.137984053
01-Aug-07	30-Aug-07	4,600	0.198280889	0.154485538	25,647	1.649510342	4.054570100
01-Aug-07	30-Aug-07	4,650	0.193998608	0.157726943	4,129	1.386723803	5.506102777
01-Aug-07	30-Aug-07	4,700	0.193757333	0.164291497	10,814	1.181957805	4.921665971

Appendix B

Error Sum of Squares (Both Approaches) for the Days on Which Option Prices are Predicted Based on Last Day's Data							
		Black-Scholes Method		Modified Black-Scholes Method			
Date	Expiry	Error	Volatility	Error	Volatility	Skewness	Kurtosis
02-Aug-07	30-Aug-07	60927.049420	0.185121765	97.32835152	0.153913651	2.133776387	–2.441662433
06-Aug-07	30-Aug-07	39604.399560	0.173739545	914.33976520	0.160598894	1.210660684	–4.225859386
08-Aug-07	30-Aug-07	38326.326700	0.196681537	571.47327930	0.183089493	1.271304849	–2.984777628
10-Aug-07	30-Aug-07	30481.284120	0.213455491	898.20771310	0.163583530	1.028276584	–4.059784161
14-Aug-07	30-Aug-07	13291.718490	0.237645487	459.52811120	0.204639867	0.950000204	–0.195634945
17-Aug-07	30-Aug-07	5052.086997	0.285664629	4257.80414800	0.213343235	0.304324055	0.216262002
21-Aug-07	30-Aug-07	1991.894706	0.313496030	1041.29920200	0.305517736	0.297634462	2.867832614
23-Aug-07	30-Aug-07	2495.495556	0.324967867	1201.37730800	0.318174196	–0.142758840	4.630263535
27-Aug-07	30-Aug-07	4477.808672	0.252838284	2055.11056900	0.300983547	–0.030773994	3.524009033
29-Aug-07	30-Aug-07	6759.377573	0.203734698	1528.58756700	0.221626403	0.268942068	2.639804032
02-Aug-07	27-Sep-07	24545.597200	0.191765495	1273.51681000	0.187315757	2.009737929	2.989815228
06-Aug-07	27-Sep-07	27625.473800	0.188598911	495.58612340	0.171705501	1.997653383	3.003895318
08-Aug-07	27-Sep-07	14389.475240	0.206241620	134.94067740	0.172402576	1.270817244	–1.367706694
10-Aug-07	27-Sep-07	30918.056620	0.208012031	1416.43584600	0.152661136	–0.073796567	–2.463725386
14-Aug-07	27-Sep-07	14957.315910	0.227459773	540.24060740	0.203200152	0.410949443	–0.211781326
17-Aug-07	27-Sep-07	7883.952548	0.279622797	2727.78576800	0.331290893	–0.652350316	10.86069605
21-Aug-07	27-Sep-07	5047.711927	0.289356020	637.11129780	0.322711979	–0.493129007	4.556720639

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Appendix B

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Error Sum of Squares (Both Approaches) for the Days on Which Option Prices are Predicted Based on Last Day's Data							
		Black-Scholes Method		Modified Black-Scholes Method			
Date	Expiry	Error	Volatility	Error	Volatility	Skewness	Kurtosis
23-Aug-07	27-Sep-07	9241.406586	0.269242499	1369.384506000	0.357319115	–0.876827836	7.276004416
27-Aug-07	27-Sep-07	13192.936780	0.241965057	928.763302900	0.355687202	–0.977988769	8.576923242
29-Aug-07	27-Sep-07	19683.251300	0.212769884	1729.637639000	0.690747595	–2.266262029	17.21665822
31-Aug-07	27-Sep-07	50487.608870	0.187153381	2617.945979000	0.186128974	1.051971388	–0.494280462
04-Sep-07	27-Sep-07	79457.122840	0.187578787	2893.348446000	0.174480048	0.465772816	–.484379269
06-Sep-07	27-Sep-07	47443.876770	0.168655006	3376.076400000	0.162594612	1.041245525	–2.644880322
10-Sep-07	27-Sep-07	46967.914050	0.169294233	913.948261100	0.171923206	–0.110640111	1.387471659
12-Sep-07	27-Sep-07	28298.007610	0.211085038	371.140451300	0.206896848	–0.339116563	2.285022148
14-Sep-07	27-Sep-07	34601.814090	0.218454552	260.334711600	0.201178654	–0.150590446	0.523953447
18-Sep-07	27-Sep-07	30979.721380	0.254671810	975.456265200	0.212364379	–0.684492897	2.893715221
20-Sep-07	27-Sep-07	40782.523790	0.237607120	10487.036380000	0.281737588	–0.940859816	7.669125836
24-Sep-07	27-Sep-07	66787.286640	0.230107453	1587.041933000	0.174691159	0.206531190	0.47705759
26-Sep-07	27-Sep-07	25354.982340	0.313028904	4921.836667000	0.196094293	0.185791029	1.278697089
06-Aug-07	25-Oct-07	13572.334570	2.26909E-24	54.427758680	0.182696846	0.886052145	3.047513507
27-Aug-07	25-Oct-07	34.31395316	0.235429191	49.133585290	0.318038088	–0.818132226	1.740351362
30-Aug-07	25-Oct-07	12907.078540	0.201130594	15.560568920	0.305265361	2.260098004	2.837924755
03-Sep-07	25-Oct-07	8358.235683	0.189943518	78.778739650	0.111195223	–2.491345330	–16.92834634
05-Sep-07	25-Oct-07	15024.163260	0.182693215	113.800967600	0.189859232	–0.300374473	2.08864019
07-Sep-07	25-Oct-07	15512.359630	0.179626450	16.742904910	0.162033309	–0.150605651	–0.410715423
11-Sep-07	25-Oct-07	16588.198450	0.198409735	1186.875185000	0.182954322	0.508223368	4.764283899
13-Sep-07	25-Oct-07	17295.417290	0.194706798	325.615247800	0.297277594	–0.983474273	10.07140551
17-Sep-07	25-Oct-07	23140.151570	0.193442793	9.261525815	0.185361413	–0.270486098	1.09183015
19-Sep-07	25-Oct-07	42622.065200	0.210323665	1043.873968000	0.186192854	0.223479441	–0.515184411
21-Sep-07	25-Oct-07	49777.506880	0.187335924	3959.685787000	0.206367020	0.845542987	7.771007336
25-Sep-07	25-Oct-07	26850.467770	0.220757391	3632.730714000	0.630504493	1.636151396	13.64128097
01-Oct-07	25-Oct-07	51159.549590	0.236802820	652.095391700	0.230711499	–0.891869967	–0.300833878
04-0ct-07	25-Oct-07	16827.839740	0.257052229	3783.990284000	0.227174056	–0.564526729	2.663369987
08-0ct-07	25-Oct-07	4619.269149	0.308464606	3270.476348000	0.281560630	–0.874079836	3.624433127
10-0ct-07	25-Oct-07	15873.111130	0.316287833	14587.748100000	0.310515665	–0.669919981	2.314873149
12-0ct-07	25-Oct-07	10227.909740	0.350443090	9994.201691000	0.304810038	–0.652502455	5.641813353
16-0ct-07	25-Oct-07	9071.745112	0.382296023	6002.896840000	0.367113463	–1.537935272	4.662973066
18-0ct-07	25-Oct-07	10575.534590	0.407934706	11910.813170000	0.313228748	–0.567909754	–0.003820752
22-Oct-07	25-Oct-07	18142.162010	0.506001568	6954.726385000	0.373287352	0.034676262	3.557241188
05-Sep-07	29-Nov-07	4432.286627	0.170201607	496.739735900	0.232568702	2.185911954	2.813007118
11-Sep-07	29-Nov-07	1.26963E-18	13338.55904	0	0.188072868	2.011495679	2.987036955

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Error Sum of Squares (Both Approaches) for the Days on Which Option Prices are Predicted Based on Last Day's Data							
		Black-Scholes Method		Modified Black-Scholes Method			
Date	Expiry	Error	Volatility	Error	Volatility	Skewness	Kurtosis
14-Sep-07	29-Nov-07	4210.1981370	0.197395971	1057.31413500	0.16114617	1.820778937	3.187688372
Ol-Oct-07	29-Nov-07	88.8369657	0.220402452	88.71934226	0.201692046	–0.790082127	–0.076654269
04-0ct-07	29-Nov-07	330.4163573	0.250588352	77.65432153	0.189138562	–1.024863272	–1.073092029
12-0ct-07	29-Nov-07	680.0303735	0.291539505	209.29181000	0.435122662	–0.501176589	9.865252808
16-0ct-07	29-Nov-07	1450.4415290	0.331122876	825.56376780	0.381138556	–0.596131803	3.940308769
18-0ct-07	29-Nov-07	1527.8950540	0.364658683	894.83217570	0.349596330	–1.196699546	0.795316114
22-Oct-07	29-Nov-07	5968.1208590	0.387403218	3168.10033500	0.813332739	–1.497275590	12.22853378
24-Oct-07	29-Nov-07	16968.2785800	0.345191171	11278.17524000	0.626406123	–0.796780204	5.925996473

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